**Improper Integral**

**Def:** If either a**ny** one of (or both) the limit of the integral is (or are ) infinite or is infinitely discontinuous or indeterminate at *a* (lower limit) or *b* (upper limit ) or both or at any one or more points between *a* and *b* , then the integral is called an improper integral or infinity integral. The improper integral can be classified into three kinds

1. **Infinite range** : The limit of the integral either lower or upper or both limits are infinite. This type of integrals can be written as
2. provided is integrable in (*a,b*) and this limit exists.
3. provided is integrable in (*a,b*) and this limit exists.

and c arbitrary , provided is integrable in (*a,b*) and this limit exists

Evaluate1

Soln Here upper limit is infinity

Evaluate2

Soln Here lower limit is infinity . So this

integral can be written as

Evaluate3

Soln Here upper and lower limit both are infinity . So this

integral can be written as

+

=

=

=

1. **Integrand Infinitely discontinuous at a point** :

is infinitely discontinuous at lower limit*a,*

*i,e* if then

can be written as provided

is integrable in (*a,b*) and this limit exists.

is infinitely discontinuous at upper limit*b,*

*i,e* if then

can be written as provided

is integrable in (*a,b*) and this limit exists.

is infinitely discontinuous at an internal

point *c, ( i,e* if

then can be written as

provided

is integrable in (*a,b*) and this limit exists

is infinitely discontinuous at both upper and

lower limit*a*and*b,*

*i,e* if then

can be written as

and c arbitrary , where two integrals exists

Evaluate3

Soln Here as So this

integral can be written as

=

Evaluate4

Soln Here as So this

integral can be written as

Evaluate5

Soln Here as So this

integral can be written as

**Convergence and divergence of improper integral**

If the value of exists that is its value is finite then the integral is convergent , otherwise it is divergent.

Example: Examine convergence of

Soln.

Since is finite , so is convergent.

Example: Examine convergence of

Soln.

Since is not exist , so is divergent.

**Test for the convergence and divergence**

**First kind convergence**: The improper integrals is first kind when the range of integration is infinity but integrand bound .

1. **Comparison test:**
2. If and is convergent then is also convergent
3. If and is divergent then is also divergent

Example: Examine the convergent of

Solution: Let f and Let

since

Now

Since is finite . So is convergent .

But . Hence by comparison test

convergent

Example: Examine the convergent of

Solution: Let f and Let

Now

Since is not exist . So is divergent .

But . Hence by comparison test

divergent

1. **The test**

Let function be bounded and integrable in the interval when .

1. If there is a number such that  exist finitely , the limit being neither zero nor infinite , then convergent.
2. If there is a number such that exist and is not zero , then divergent

The value of is easily taken to be under highest power of *x* in denominator(D) minus highest power of x in numerator(N) in

Example: Examine the convergent of

Solution: Let f

,

Since and  **.** So given integral is divergent

Example: Examine the convergent of

Solution: Let f

,

Since and  **.** So given integral is convergent

**Second kind convergence**: The improper integrals is first kind When the integrals is infinity and integration is finite i.,e . or or

,

1. **Comparison test:**

If finite i.,e. neither zero or infinity , then two integrals and either convergent or both divergent

If is convergent when and divergent if

At is the point of infinity discontinuities then let

Example: Examine the convergent of

Solution: Let f and Let

since

Now

Since is finite . So is convergent .

But . Hence by comparison test

convergent

Example: Examine the convergent of

Solution: Let f and Let

Now

Since is not exist . So is divergent .

But . Hence by comparison test

divergent

1. **The test**

Let function be bounded and integrable in the interval when .

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